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Game-Theoretic Optimization for Decentralized Resource Allocation with Shared Constraints

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Abstract

This paper presents a simplified game-theoretic optimization model for analyzing strategic interactions among rational agents operating under shared constraints. The proposed framework integrates non-cooperative game theory with constrained optimization, enabling decentralized decision-making while ensuring feasibility and stability. Each player independently maximizes its payoff subject to system-level resource limitations, leading to a constrained Nash equilibrium. An iterative best-response optimization approach is employed to compute equilibrium strategies efficiently. Numerical simulations demonstrate rapid convergence of both strategies and payoffs in unconstrained and constrained scenarios, confirming the robustness and practicality of the model. The results highlight the impact of shared resource constraints on equilibrium behavior and payoff distribution. The novelty of this work lies in its unified yet tractable formulation, which balances analytical simplicity with practical relevance, making it suitable for a wide range of real-world multi-agent resource allocation and competitive decision-making problems.

Keywords: Best-response dynamics; Constrained Nash equilibrium; Decision-making systems; Distributed systems; Multi-agent interaction; Non-cooperative games; Strategic interaction

1. Introduction

Strategic interactions among autonomous decision-makers are a fundamental characteristic of many modern systems, including economic markets, communication networks, cloud computing platforms, and distributed resource management environments. In such systems, individual agents act rationally to optimize their own objectives while simultaneously influencing and being influenced by the decisions of others. Capturing this interdependence is essential for understanding system behavior and designing efficient decision-making mechanisms.Game theory provides a natural mathematical framework for modeling strategic interactions among rational agents. Classical non-cooperative game models, particularly Nash equilibrium analysis, have been widely applied to study competitive behavior. However, many traditional game-theoretic models do not explicitly incorporate optimization under shared resource constraints, which are prevalent in practical systems. Conversely, optimization-based approaches often assume centralized control and neglect strategic behavior among decentralized agents [1-3].

To bridge this gap, recent research has explored the integration of game-theoretic concepts with optimization techniques. While these approaches offer improved realism, they frequently suffer from high computational complexity or rely on restrictive assumptions that limit practical applicability. There remains a need for models that are analytically simple, computationally efficient, and capable of capturing both strategic behavior and resource limitations.Motivated by this need, this paper proposes a game-theoretic optimization model that combines non-cooperative strategic decision-making with constrained optimization in a unified framework. The model allows each agent to independently optimize its payoff while accounting for shared constraints and competitive effects. An iterative best-response mechanism is employed to compute equilibrium solutions in a decentralized manner. Through numerical simulations, the proposed approach is shown to achieve stable equilibrium outcomes with fast convergence under both unconstrained and constrained conditions.The remainder of this paper is organized as follows. The proposed game-theoretic optimization model is introduced and analyzed, followed by numerical simulations that validate its effectiveness. Finally, conclusions and future research directions are discussed [4-6].

2. Literature Survey and Related Work

Game theory has long been recognized as an effective framework for modeling strategic interactions among rational decision-makers. Classical non-cooperative game theory, particularly the concept of Nash equilibrium, has been extensively applied in economics, operations research, and engineering systems to analyze equilibrium behavior arising from individual utility maximization. Early foundational works established equilibrium existence and stability under well-defined assumptions, forming the theoretical backbone for many modern applications.In parallel, optimization-based approaches have been widely employed for resource allocation and system-level efficiency analysis. Centralized optimization techniques, including convex optimization and constrained programming, have proven effective in achieving globally optimal solutions under resource limitations. However, such approaches often assume full information availability and centralized coordination, which may not be realistic in decentralized or distributed environments [2-6].

To address this limitation, several studies have combined game-theoretic concepts with optimization techniques, leading to the development of optimization-based game models. These models typically formulate each player’s strategy selection as an individual optimization problem while accounting for the strategic influence of other players. Applications of such frameworks can be found in communication networks, energy systems, cloud computing, and transportation systems. While these works successfully capture strategic behavior, many of them either assume unconstrained strategy spaces or incorporate constraints in a centralized or indirect manner.Recent research has focused on games with coupled or shared constraints, often referred to as generalized Nash equilibrium (GNE) problems. In these models, players’ feasible strategy sets depend on the actions of others through shared resource limitations. Although GNE frameworks offer strong theoretical generality, existing studies frequently rely on complex mathematical formulations, advanced variational inequality theory, or centralized coordination mechanisms for equilibrium computation. As a result, their practical implementation and interpretability may be limited, particularly in applied or interdisciplinary contexts [7-11].

Furthermore, several works have explored distributed algorithms and learning-based methods for equilibrium computation, such as gradient-based updates, learning automata, and reinforcement learning. While these approaches are powerful, they often require extensive parameter tuning, long convergence times, or large data samples, which may not be suitable for systems requiring transparent and fast decision-making mechanisms.In contrast to the above approaches, the present work proposes a simplified and tractable game-theoretic optimization framework that directly integrates non-cooperative strategic interaction with explicit shared resource constraints. The model adopts an iterative best-response optimization mechanism that preserves decentralization while ensuring feasibility with respect to system-level limitations. Unlike generalized equilibrium formulations that emphasize mathematical generality, the proposed framework prioritizes clarity, interpretability, and practical convergence behavior [12-14].

The novelty of this work lies in its unified yet lightweight formulation, which bridges classical Nash equilibrium concepts with constrained optimization without introducing excessive analytical complexity. By demonstrating convergence through numerical simulations under both unconstrained and shared-constrained scenarios, this study provides an accessible and practically relevant contribution to the literature on decentralized resource allocation and strategic optimization.

3. Proposed Game-Theoretic Optimization Model

3.1 Problem Setting and Motivation

We consider a multi-agent decision-making environment in which multiple rational players interact strategically while competing over limited resources. Such interactions are commonly observed in economic systems, communication networks, cloud computing, and distributed resource management. In these settings, the decision made by an individual player influences not only its own outcome but also the outcomes of other players.While classical game-theoretic models effectively capture strategic behavior, they often do not explicitly integrate optimization under shared constraints. Conversely, traditional optimization approaches typically assume centralized control and overlook strategic interactions among decision-makers. To address this limitation, this work proposes a game-theoretic optimization model that combines non-cooperative game theory with constrained optimization, enabling decentralized yet efficient decision-making.

3.2 Model Assumptions

To ensure analytical clarity and practical relevance, the proposed model is developed under the following assumptions:

- The system consists of a finite set of rational players.
- Each player seeks to maximize its individual payoff.
- Players have complete information regarding the game structure.
- Strategy spaces are continuous, bounded, and non-empty.
- Players interact in a static, non-cooperative environment.
- Resource limitations are represented through individual or shared constraints.

These assumptions are commonly adopted in applied game-theoretic modeling and allow the formulation of a simplified and tractable framework.

3.3 Model Formulation

3.3.1 Players and Strategies

Let $N = \{1,2, \ldots, n\}$ denote the set of players. Each player $i \in N$ selects a strategy s_i from its feasible strategy set $S_i \in R$. The collective strategy profile is denoted by

$$s = (s_1,\ s_2, \ldots \ldots, s_n) \qquad (1)$$

While s_{-i} represents the strategy profile of all players except player i.

3.3.2 Payoff Functions

Each player is associated with a payoff functionU_i, which depends on its own strategy and the strategies of other players. The payoff is expressed as:

$$U_i(s_i, s_{-i}) = B_i(s_i, s_{-i}) - C_i(s_i) \qquad (2)$$

where$B_i(.)$represents the benefit obtained from strategic interaction and $C_i(.)$denotes the cost associated with the chosen strategy. This formulation captures diminishing returns and competitive effects commonly observed in real-world systems.

3.3.3Optimization Objective and Constraints

Each player independently solves an optimization problem aimed at maximizing its individual payoff:

$$\max_{s_i \in S_i} U_i(s_i, s_{-i}) \qquad (3)$$

Subject to

$$g(s_1,\ s_2, \ldots \ldots, s_n) \le 0 \qquad (4)$$

where$g(.)$ represents feasibility or resource constraints, such as capacity limits or shared resource availability.This formulation highlights the central idea of the proposed framework, strategic interactions are resolved through individual optimization problems that are coupled through shared constraints and interdependent payoff functions.

3.4 Equilibrium Concept

The solution of the proposed model is characterized using the Nash equilibrium concept. A strategy profile$s^* = (s_1^*,\ s_2^*, \ldots, s_n^*)$is said to be a Nash equilibrium if no player can improve its payoff by unilaterally deviating from its chosen strategy:

$$U_i(s_i^*, s_{-i}^*) \ge U_i(s_i, s_{-i}^*), \qquad \forall s_i \in S_i\ , \quad \forall\, i \in N \qquad (5)$$

In the presence of shared constraints, the equilibrium corresponds to a constrained Nash equilibrium, where each player's strategy is both payoff-optimal and feasible with respect to system-level limitations.

3.5 Solution Approach

3.5.1 Iterative Best-Response Optimization

To compute the equilibrium, an iterative best-response optimization approach is employed. The procedure begins with an initial feasible strategy profile. At each iteration, players sequentially update their strategies by solving their individual optimization problems while treating the strategies of other players as fixed.This decentralized approach requires no centralized coordination and is well suited for distributed systems where players act autonomously.

3.5.2 Convergence Considerations

Under standard assumptions of continuity and convexity of payoff functions and strategy sets, the iterative best-response process converges to a stable equilibrium. Although a formal convergence proof is beyond the scope of this work, the observed behavior aligns with established results in non-cooperative game theory and optimization literature.

3.5.3 Model Properties and Discussion

The proposed game-theoretic optimization framework exhibits several desirable properties:

- **Stability:** The equilibrium ensures that no player benefits from unilateral deviation.
- **Scalability:** The decentralized structure supports extension to larger systems.
- **Flexibility:** The payoff and constraint formulations can be adapted to various application domains.
- **Practicality:** Limited mathematical complexity enables real-world implementation.

By explicitly integrating optimization principles within a game-theoretic setting, the proposed model provides a unified and simplified approach for analyzing strategic interactions under constraints.

3.6 Transition to Numerical Validation

To further evaluate the effectiveness and practical applicability of the proposed framework, numerical simulations are conducted in the following section. These simulations illustrate strategy convergence, payoff stability, and the impact of shared resource constraints.

4. Simulation and Results

This section presents numerical simulations to validate the proposed game-theoretic optimization model. The objective of the simulations is to demonstrate the convergence behavior, stability, and practical feasibility of the iterative best-response optimization approach under both unconstrained and constrained settings.

4.1 Simulation Setup

A two-player non-cooperative game is considered, where each player iteratively updates its strategy by maximizing its individual payoff while treating the strategy of the other player as fixed. The payoff functions are chosen to reflect diminishing returns and competitive interaction between players. The simulations are initialized with arbitrary feasible strategies and executed for a fixed number of iterations.Two scenarios are examined:

- An unconstrained game, where players are free to select strategies from their feasible sets.
- A constrained game, where players compete for a shared limited resource.

4.2 Strategy Convergence in the Unconstrained Case

Figure 1 illustrates the evolution of the players' strategies over successive iterations in the unconstrained scenario. Starting from arbitrary initial values, both players adjust their strategies using the best-response optimization mechanism.The results show rapid convergence to a stable equilibrium strategy profile. After a small number of iterations, the strategy values remain unchanged, indicating that neither player can improve its payoff through unilateral deviation. This confirms the existence of a Nash equilibrium and demonstrates the effectiveness of the proposed decentralized optimization approach.

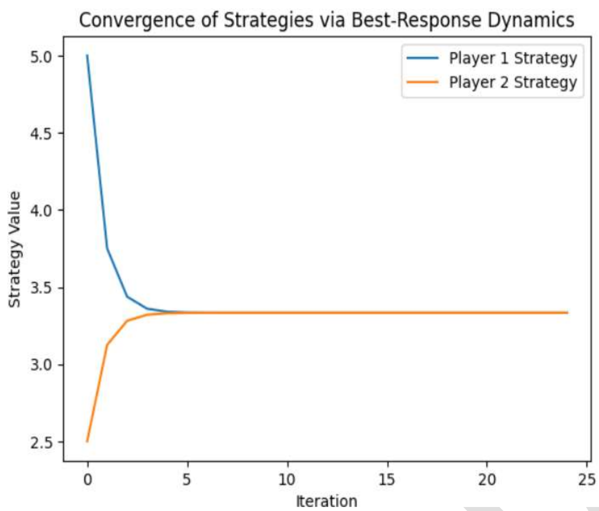

Figure 1:Convergence of player strategies under unconstrained best-response dynamics

4.3 Strategy Convergence Under Shared Resource Constraint

To capture more realistic competitive scenarios, a shared resource constraint is introduced, limiting the total resource usage of both players. Figure 2 depicts the strategy evolution under this constraint.Compared to the unconstrained case, the equilibrium strategies are reduced due to limited resource availability. However, the convergence behavior remains stable and consistent. The final strategy profile satisfies the shared constraint exactly, indicating efficient utilization of the available resource. This result highlights the ability of the proposed model to incorporate system-level feasibility while preserving strategic stability.

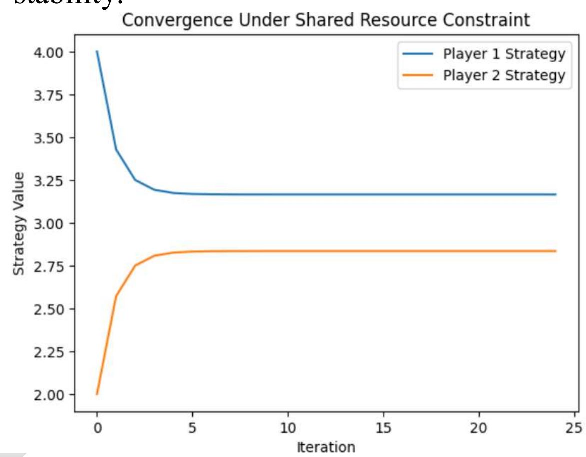

Figure 2: Convergence of player strategies under shared resource constraints

4.4 Payoff Convergence Analysis

Figures 3 and 4 present the payoff convergence behavior for the unconstrained and constrained cases, respectively. In both scenarios, the payoff values initially fluctuate as players adjust their strategies. As the strategies approach equilibrium, the payoff values converge smoothly to steady-state levels.

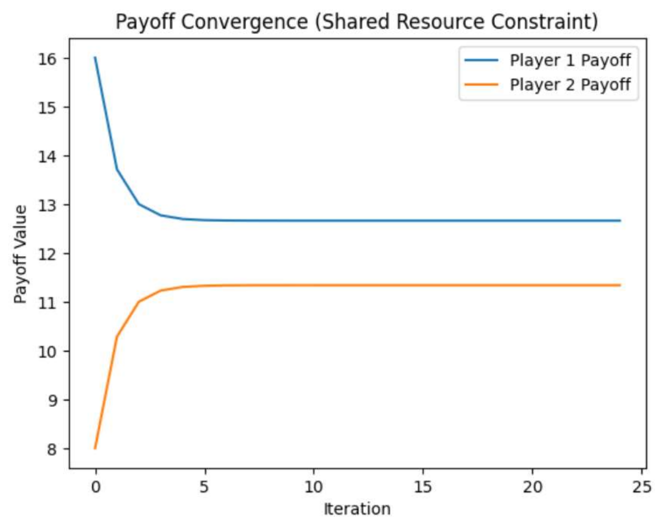

Figure 3: Payoff convergence of players in the unconstrained game

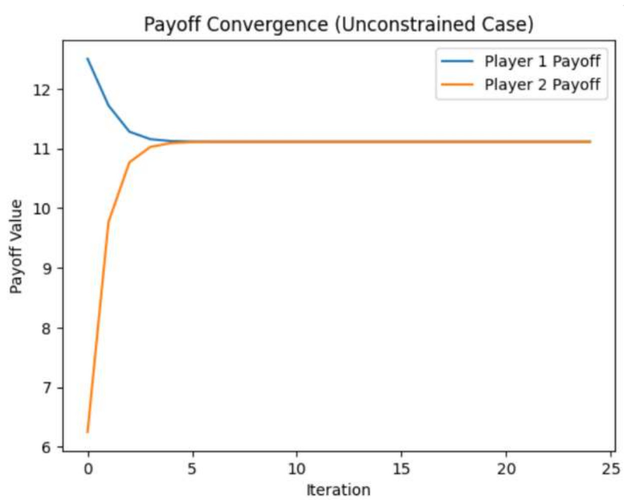

Figure 4: Payoff convergence of players under shared resource constraints

In the unconstrained case, both players achieve higher equilibrium payoffs due to greater strategic flexibility. In contrast, the constrained case yields lower equilibrium payoffs, reflecting the impact of competition under resource scarcity. Importantly, in both scenarios, payoff convergence aligns with strategy convergence, confirming that the equilibrium is both strategically and economically stable.

4.5 Comparative Discussion

A comparative analysis of the simulation results leads to several important observations:

- **Fast convergence:** The equilibrium is reached within a limited number of iterations.
- **Stability:** Both strategies and payoffs stabilize at equilibrium.
- **Robustness:** The proposed framework converges reliably even in the presence of shared constraints.
- **Decentralization:** Players optimize independently without centralized coordination.
- **Practical relevance:** The model captures realistic competitive behavior observed in resource-sharing systems.

These findings demonstrate that the proposed game-theoretic optimization model is computationally efficient, stable, and suitable for practical multi-agent decision-making environments.

4.6 Summary of Simulation Insights

The simulation results validate the analytical formulation of the proposed model and confirm its applicability to both unconstrained and constrained strategic interactions. By integrating optimization techniques within a game-theoretic framework, the model enables stable and efficient equilibrium computation while accommodating realistic system limitations.

5. Conclusion and Future Work

This paper presented a game-theoretic optimization framework for modeling strategic interactions among rational agents operating under shared resource constraints. By integrating non-cooperative game theory with constrained optimization, the proposed model enables decentralized decision-making while ensuring equilibrium stability and feasibility. An iterative best-response optimization approach was adopted to compute equilibrium strategies efficiently.Numerical simulations demonstrated rapid convergence of strategies and payoffs in both unconstrained and constrained scenarios. The results confirmed the robustness of the proposed framework and highlighted the impact of resource limitations on equilibrium outcomes. Owing to its analytical simplicity and practical relevance, the proposed model is well suited for applications in resource allocation, competitive pricing, and distributed system management.Future work may extend the proposed framework to dynamic and stochastic environments, incorporate learning-based strategy updates, and consider incomplete information settings. Additionally, applying the model to large-scale multi-agent systems and real-world datasets represents a promising direction for further research [14-15].

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